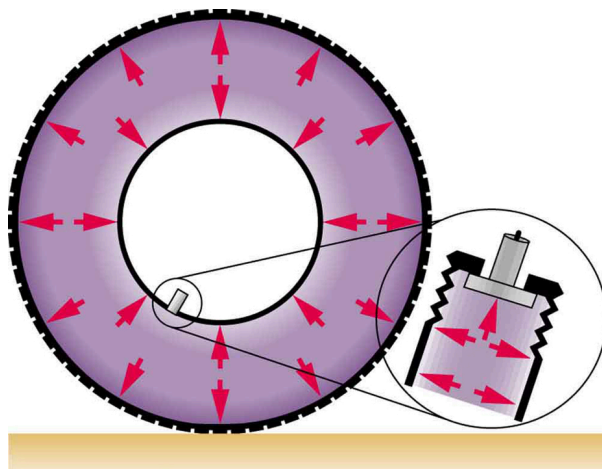
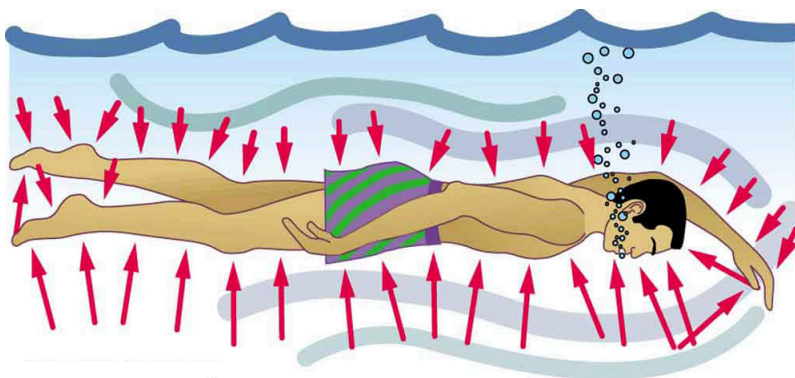


pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [Figure 11.8.](#))



**Figure 11.7** Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



**Figure 11.8** Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.

### Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other. [Click to open media in new browser. \(https://phet.colorado.edu/en/simulation/legacy/gas-properties\)](https://phet.colorado.edu/en/simulation/legacy/gas-properties)

## 11.4 Variation of Pressure with Depth in a Fluid

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [Figure 11.9](#). Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid  $mg$  divided by the area  $A$  supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}. \quad 11.12$$

We can find the mass of the fluid from its volume and density:

$$m = \rho V. \quad 11.13$$

The volume of the fluid  $V$  is related to the dimensions of the container. It is

$$V = Ah, \quad 11.14$$

where  $A$  is the cross-sectional area and  $h$  is the depth. Combining the last two equations gives

$$m = \rho Ah. \quad 11.15$$

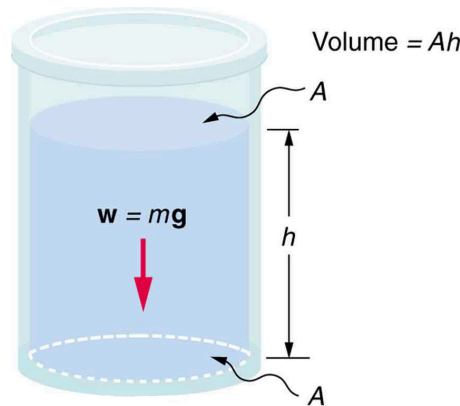
If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho Ah)g}{A}. \quad 11.16$$

The area cancels, and rearranging the variables yields

$$P = h\rho g. \quad 11.17$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation  $P = h\rho g$  represents the pressure due to the weight of any fluid of *average density*  $\rho$  at any depth  $h$  below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. [Example 11.4](#) illustrates this situation.



**Figure 11.9** The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

### **EXAMPLE 11.3**

#### **Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?**

In [Example 11.1](#), we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [Figure 11.10](#).) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be  $1.96 \times 10^{13}$  N).

#### **Strategy for (a)**

The average pressure  $\bar{P}$  due to the weight of the water is the pressure at the average depth  $\bar{h}$  of 40.0 m, since pressure increases linearly with depth.

**Solution for (a)**

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g. \quad 11.18$$

Entering the density of water from [Table 11.1](#) and taking  $\bar{h}$  to be the average depth of 40.0 m, we obtain

$$\begin{aligned} \bar{P} &= (40.0 \text{ m}) \left( 10^3 \frac{\text{kg}}{\text{m}^3} \right) \left( 9.80 \frac{\text{m}}{\text{s}^2} \right) \\ &= 3.92 \times 10^5 \frac{\text{N}}{\text{m}^2} = 392 \text{ kPa}. \end{aligned} \quad 11.19$$

**Strategy for (b)**

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A. \quad 11.20$$

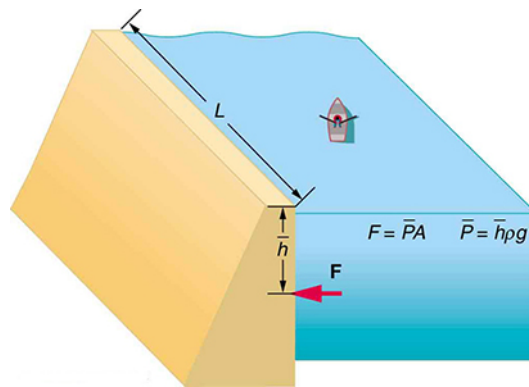
**Solution for (b)**

We have already found the value for  $\bar{P}$ . The area of the dam is  $A = 80.0 \text{ m} \times 500 \text{ m} = 4.00 \times 10^4 \text{ m}^2$ , so that

$$\begin{aligned} F &= (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2) \\ &= 1.57 \times 10^{10} \text{ N}. \end{aligned} \quad 11.21$$

**Discussion**

Although this force seems large, it is small compared with the  $1.96 \times 10^{13} \text{ N}$  weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

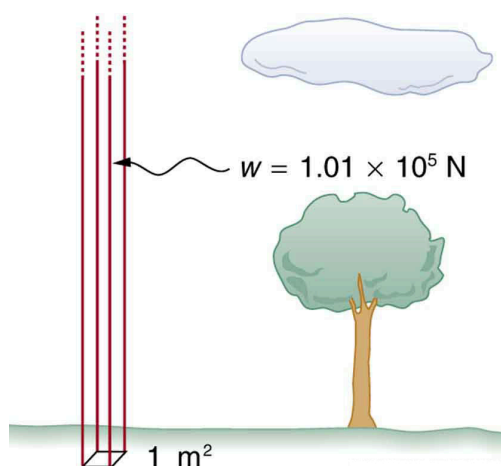


**Figure 11.10** The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

*Atmospheric pressure* is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather “highs” and “lows”). However, the average pressure at sea level is given by the *standard atmospheric pressure*  $P_{\text{atm}}$ , measured to be

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2 = 101 \text{ kPa}. \quad 11.22$$

This relationship means that, on average, at sea level, a column of air above  $1.00 \text{ m}^2$  of the Earth's surface has a weight of  $1.01 \times 10^5 \text{ N}$ , equivalent to 1 atm. (See [Figure 11.11](#).)



**Figure 11.11** Atmospheric pressure at sea level averages  $1.01 \times 10^5$  Pa (equivalent to 1 atm), since the column of air over this  $1 \text{ m}^2$ , extending to the top of the atmosphere, weighs  $1.01 \times 10^5 \text{ N}$ .

### EXAMPLE 11.4

#### Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in [Table 11.1](#).

##### Strategy

If we solve  $P = h\rho g$  for density, we see that

$$\bar{\rho} = \frac{P}{hg}.$$

11.23

We then take  $P$  to be atmospheric pressure,  $h$  is given, and  $g$  is known, and so we can use this to calculate  $\bar{\rho}$ .

##### Solution

Entering known values into the expression for  $\bar{\rho}$  yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(120 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} = 8.59 \times 10^{-2} \text{ kg/m}^3.$$

11.24

##### Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in [Table 11.1](#) as  $1.29 \text{ kg/m}^3$ —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

### EXAMPLE 11.5

#### Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

##### Strategy

We begin by solving the equation  $P = h\rho g$  for depth  $h$ :

$$h = \frac{P}{\rho g}.$$

11.25

Then we take  $P$  to be 1.00 atm and  $\rho$  to be the density of the water that creates the pressure.

### Solution

Entering the known values into the expression for  $h$  gives

$$h = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m}.$$

11.26

### Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in [Pascal's Principle](#) that fluid pressures always add in this way.

## 11.5 Pascal's Principle

**Pressure** is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

### Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

### Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in [Figure 11.12](#).